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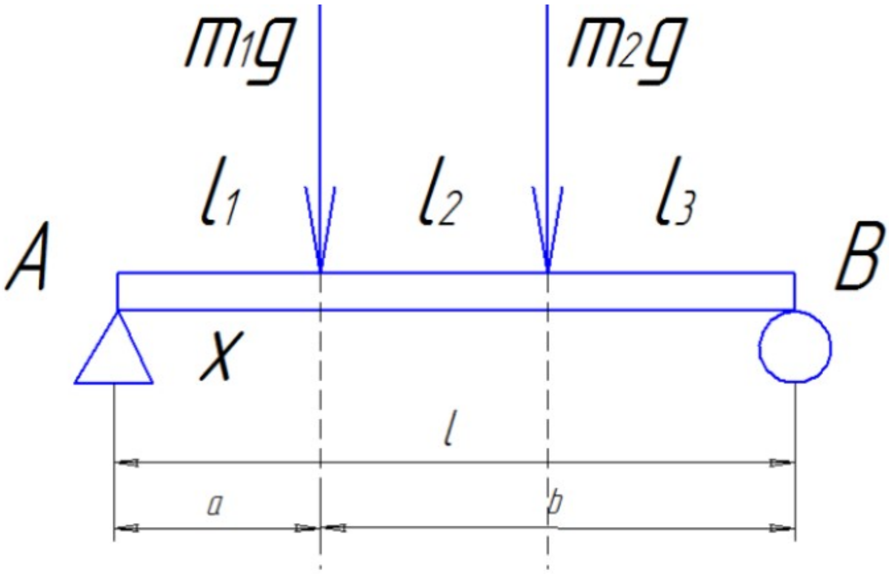
Vibration problem:

Task:

A flywheel of mass m_1 and a pulley of mass m_2 are to be mounted on a shaft length l (see Figure 1). Determine their locations l_1 and l_2 in order to maximize the fundamental frequency of vibration of the system. If m_1 = 100 kg, m_2 = 50 kg and l = 2 m.



Solution:

Basic relationship:

$$w(x)=\begin{cases}\dfrac{Pbx}{6EIl}\left(l^2-b^2-x^2\right); & 0\le x\le a\\ \dfrac{-Pa(l-x)}{6EIl}\left(a^2+x^2-2lx\right); & a\le x\le b\end{cases}$$

Deflection of mass m_1 due to load m_1g:

Using $x = l_1$, $b = l - l_1$ and $l = 2\ m$:

$$w'_1=\frac{(100\cdot 9{,}81)(2-l_1)l_1}{6EI\cdot 2}\left[4-(2-l_1)^2-l_1^2\right]=\frac{981\cdot l_1^2(2-l_1)^2}{6EI}$$

Deflection of mass m_2 due to load m_1g:

Using $x = l_1 + l_2$, $a = l_1$, $b = 2 - l_1$ and $l = 2\ m$:

$$w'_2 = -\frac{(100\cdot 9{,}81)l_1(2-l_1-l_2)}{6EI\cdot 2}\left[l_1^2+(l_1+l_2)^2-2\cdot 2\cdot(l_1+l_2)\right] =$$
$$= -\frac{981\cdot l_1(2-l_1-l_2)}{12EI}\left(2l_1^2+l_2^2+2l_1l_2-4l_1-4l_2\right)$$

Deflection of mass m_1 due to load m_2g:

Using $x = l_1$, $l = 2\ m$, $b = (2 - l_1 - l_2)$:

$$w''_1 = \frac{(50\cdot 9{,}81)(2-l_1-l_2)l_1}{6EI\cdot 2}\left[4-(2-l_1-l_2)^2-l_1^2\right] =$$
$$= \frac{490\cdot 5l_1(2-l_1-l_2)\left(-2l_1^2-l_2^2+4l_1+4l_2-2l_1l_2\right)}{12EI}$$

Deflection of mass m_2 due to load m_2g:

Using $x = l_1 + l_2$, $l = 2\ m$ and $b = 2 - l_1 - l_2$:

$$w''_2 = -\frac{(50\cdot 9{,}81)(2-l_1-l_2)(l_1+l_2)}{6EI\cdot 2}\left[4-(2-l_1-l_2)^2-(l_1+l_2)^2\right] =$$
$$= -\frac{490\cdot 5(l_1+l_2)(2-l_1-l_2)}{12EI}\left(-2l_1^2-2l_2^2+4l_1+4l_2-4l_1l_2\right)$$

Total deflection of masses m_1 and m_2 are:

$$w_1 = w'_1 + w''_1 = \frac{981\cdot l_1^2(2-l_1)^2}{6EI} + \frac{490\cdot 5l_1(2-l_1-l_2)\left(-2l_1^2-l_2^2+4l_1+4l_2-2l_1l_2\right)}{12EI}$$

$$w_2 = w'_2 + w''_2 = \frac{490\cdot 5l_1(2-l_1-l_2)\left(-2l_1^2-l_2^2+4l_1+4l_2-2l_1l_2\right)}{12EI} -$$
$$-\frac{490\cdot 5(l_1+l_2)(2-l_1-l_2)}{12EI}\left(-2l_1^2-2l_2^2+4l_1+4l_2-4l_1l_2\right)$$

$$\omega = \left\{\frac{g(m_1w_1+m_2w_2)}{m_1w_1^2+m_2w_2^2}\right\}^{\frac{1}{2}} = 3\cdot 1321\left(\frac{2w_1+w_2}{2w_1^2+w_2^2}\right)^{\frac{1}{2}}$$

To maximize ω, we can maximize ω^2.

Now the problem is to find l_1 and l_2 to maximize

$$f = \left(\frac{2w_1+w_2}{2w_1^2+w_2^2}\right)$$

If we solve the equations:

$\frac{\partial f}{\partial l_1} = 0$ and $\frac{\partial f}{\partial l_2} = 0$, we will find

$l_1 = 0{,}33l$

$l_2 = 0{,}67l$