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Engineering
21 October 2016

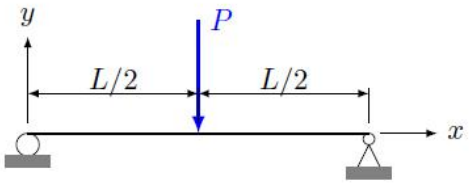


Figure 1: Simply-supported beam. The roller support on the left end provides a vertical reaction force which prevents vertical motion; the pin support on the right end provides both vertical and horizontal reaction forces, which prevent both vertical and horizontal motion. There is a point load with force P located in the middle of the beam.

The theoretical formula for the deflection is:

$$v(x) = \begin{cases} -\frac{Px}{48EI}\left(3L^2 - 4x^2\right), & 0 \le x \le L/2 \\ -\frac{P(x-L)}{48EI}\left(L^2 - 8Lx + 4x^2\right), & L/2 < x \le L \end{cases} \tag{1}$$

For your assignment, the following values are to be used:

$$\begin{aligned} E &= 4.1777 \times 10^2 \text{ GPa} \\ I &= \frac{bh^3}{12} \\ b &= 5.0 \text{ mm} \\ h &= 0.25 \text{ mm} \\ P &= Ea \\ a &= 10^{-10} \text{ m}^2 \\ L &= 2.0 \text{ cm}. \end{aligned}$$

The code:

```
clc
clear
E=417770;              % Young's modulus, MPa
h=0.25;                % Height of the beam, mm
b=5;                   % Width of the beam, mm
I=b*h^3/12;            % Inertial moment of the beam, mm^4
a=10^(-4);             % Coefficient, mm^2
A=h*b;                 % Cross area of the beam, mm^2
L=20;                  % Length of the beam, mm
P=E*a;                 % Applied force, N
x=12;
if x < L/2
   v=-P*x/(48*E*I)*(3*L^2-4*x^2);
else
   v=-P*(x-L)/(48*E*I)*(L^2-8*L*x+4*x^2);
end
v %mm
```

```
%First dereviate
if x < L/2
   f =@(y) -P*y/(48*E*I)*(3*L^2-4*y.^2);
   fprime = @(y) (12*P*y.^2-3*P*L^2)/(48*E*I); %actual derivative
of function
else
   f =@(y) -P*(y-L)/(48*E*I)*(L^2-8*L*y+4*y.^2);
   fprime = @(y) (-P*(L^2-8*L*y+4*y.^2)-P*(y-L)*(-
8*L+8*y))/(48*E*I); %actual derivative of function
end

 %step size:
 h1=0.0025;

%forward difference
dfdx_forward = (f(x+h1)-f(x))/h1
 Error_forward = fprime(x)-dfdx_forward %error

 %bacward difference
 dfdx_backward = (f(x)-f(x-h1))/h1
 Error_backward = fprime(x)-dfdx_backward %error

 %central difference
dfdx_central = (f(x+h1)-f(x-h1))/(2*h1)
 Error_central = fprime(x)-dfdx_central %error




%Second dereviate
if x < L/2
   g =@(y) -P*y/(48*E*I)*(3*L^2-4*y^2);
   fprime = @(y) (12*P*y.^2-3*P*L^2)/(48*E*I); %actual derivative
of function
else
   g =@(y) -P*(y-L)/(48*E*I)*(L^2-8*L*y+4*y^2);
   fprime2 = @(y) (-P*(L^2-8*L*y+4*y.^2)-P*(y-L)*(-
8*L+8*y))/(48*E*I); %actual derivative of function
end

 %step size:
 h=0.0025;

%forward difference
dfdx_forward2 = (g(x+h)-g(x))/h
 Error_forward2 = fprime2(x)-dfdx_forward2 %error

 %bacward difference
```

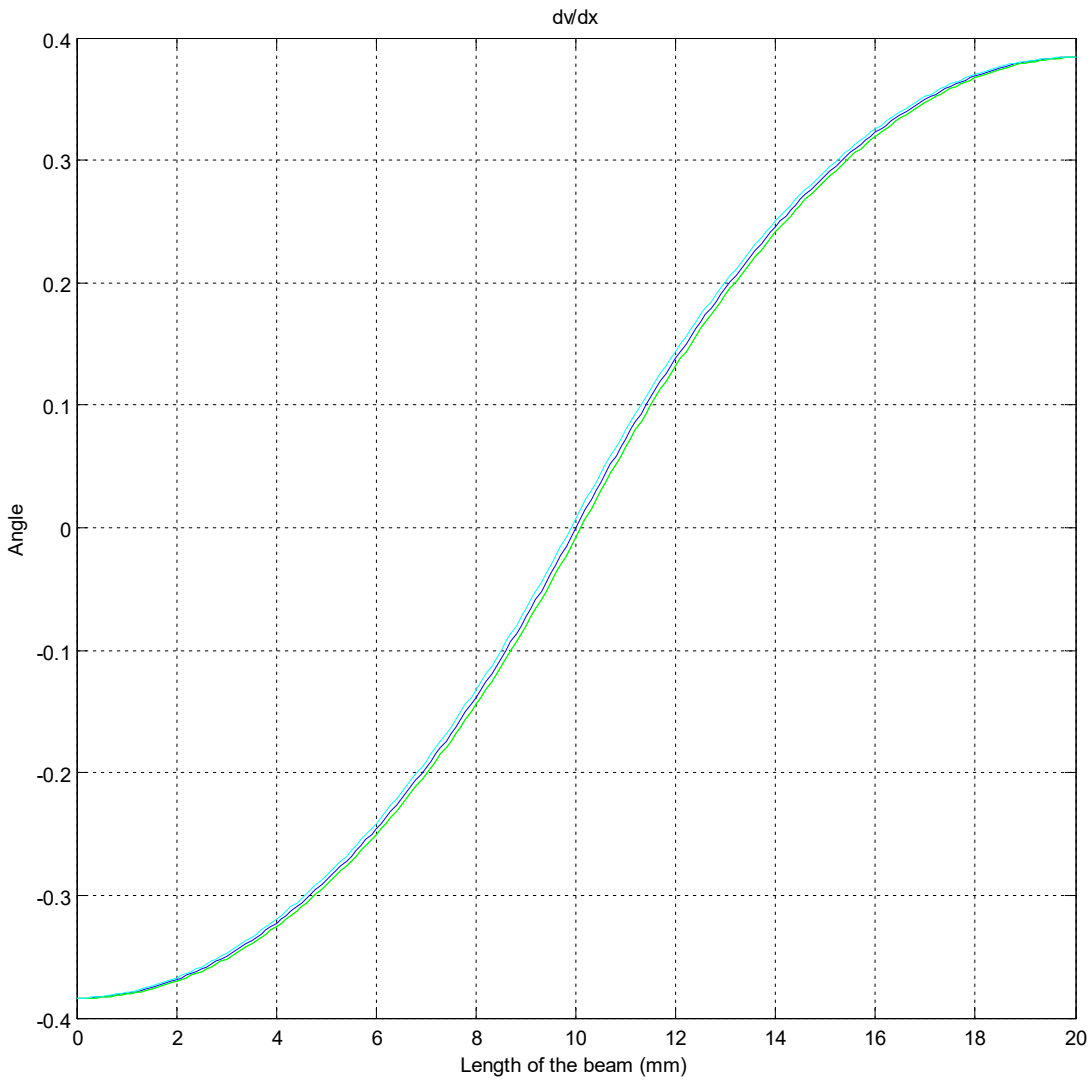
```
 dfdx_backward2 = (g(x)-g(x-h))/h
 Error_backward2 = fprime2(x)-dfdx_backward2 %error

 %central difference
dfdx_central2 = (g(x+h)-g(x-h))/(2*h)
 Error_central2 = fprime2(x)-dfdx_central2 %error


 %//////////////////////////////////////
 h1=0.1;
x=[0:0.1:10];
fprime = (12*P*x.^2-3*P*L^2)/(48*E*I); %actual derivative of
function
plot(x,fprime)
hold on
y=[10:0.1:20];
fprime2 = (-P*(L^2-8*L*y+4*y.^2)-P*((-8*L*y+8*y.^2-L*(-
8*L+8*y))))/(48*E*I); %actual derivative of function
plot(y,fprime2)
hold on

%forward difference

x=[0:0.1:10];
fprime = (12*P*(x-h1).^2-3*P*L^2)/(48*E*I); %actual derivative of
function
plot(x,fprime,'g-')
hold on
y=[10:0.1:20];
fprime2 = (-P*(L^2-8*L*(y-h1)+4*(y-h1).^2)-P*((-8*L*(y-h1)+8*(y-
h1).^2-L*(-8*L+8*(y-h1)))))/(48*E*I); %actual derivative of
function
plot(y,fprime2,'g-')
grid on
xlabel('Length of the beam (mm)')
ylabel('Angle')
title('dv/dx')
hold on

%bacward difference

x=[0:0.1:10];
dfdx_backward = (12*P*(x).^2-3*P*L^2)/(48*E*I); %actual derivative
of function
plot(x,fprime,'g-')
hold on
y=[10:0.1:20];
dfdx_backward2 = (-P*(L^2-8*L*(y)+4*(y).^2)-P*((-8*L*(y)+8*(y).^2-
L*(-8*L+8*(y)))))/(48*E*I); %actual derivative of function
```

```
plot(y,fprime2,'g-')
grid on
xlabel('Length of the beam (mm)')
ylabel('Angle')
title('dv/dx')
hold on

 %central difference
x=[0:0.1:10];
fprime = (12*P*(x+h1).^2-3*P*L^2)/(48*E*I); %actual derivative of
function
plot(x,fprime,'c-')
hold on
y=[10:0.1:20];
fprime2 = (-P*(L^2-8*L*(y+h1)+4*(y+h1).^2)-P*((-
8*L*(y+h1)+8*(y+h1).^2-L*(-8*L+8*(y+h1)))))/(48*E*I); %actual
derivative of function
plot(y,fprime2,'c-')
grid on
xlabel('Length of the beam (mm)')
ylabel('Angle')
title('dv/dx')
hold on
```

Results:

```
v =

  -2.4166


dfdx_forward =

   0.1383


Error_forward =

 -7.6792e-05


dfdx_backward =

   0.1382
```

```
Error_backward =

   7.6808e-05

dfdx_central =

    0.1382

Error_central =

   7.9999e-09

dfdx_forward2 =

    0.1383

Error_forward2 =

  -7.6792e-05

dfdx_backward2 =

    0.1382

Error_backward2 =

   7.6808e-05

dfdx_central2 =

    0.1382

Error_central2 =

   7.9999e-09
```


dv/dx
Angle
Length of the beam (mm)
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0
2
4
6
8
10
12
14
16
18
20